

2-4 Solving Equations with the Variable on Each Side

Then
 • You solved multi-step equations.

- Now**
- 1 Solve equations with the variable on each side.
 - 2 Solve equations involving grouping symbols.

Why?

• The equation $y = 1.3x + 19$ represents the number of times Americans eat in their cars each year, where x is the number of years since 1985, and y is the number of times that they eat in their car. The equation $y = -1.3x + 93$ represents the number of times Americans eat in restaurants each year, where x is the number of years since 1985, and y is the number of times that they eat in a restaurant.

The equation $1.3x + 19 = -1.3x + 93$ represents the year when the number of times Americans eat in their cars will equal the number of times Americans eat in restaurants.



Targeted TEKS
A.5(A) Solve linear equations in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides.

Mathematical Processes
A.1(E) Create and use representations to organize, record, and communicate mathematical ideas.
A.1(F) Analyze mathematical relationships to connect and communicate mathematical ideas.

New Vocabulary
 identity

1 Variables on Each Side To solve an equation that has variables on each side, use the Addition or Subtraction Property of Equality to write an equivalent equation with the variable terms on one side.



Example 1 Solve an Equation with Variables on Each Side

Solve $2 + 5k = 3k - 6$. Check your solution.

$2 + 5k = 3k - 6$	Original equation
$\quad - 3k = - 3k$	Subtract $3k$ from each side.
$2 + 2k = -6$	Simplify.
$\quad - 2 = - 2$	Subtract 2 from each side.
$2k = -8$	Simplify.
$\frac{2k}{2} = \frac{-8}{2}$	Divide each side by 2.
$k = -4$	Simplify.

CHECK $2 + 5k = 3k - 6$ Original equation

$2 + 5(-4) \stackrel{?}{=} 3(-4) - 6$ Substitution, $k = -4$

$2 + -20 \stackrel{?}{=} -12 - 6$ Multiply.

$-18 = -18$ ✓ Simplify.

Guided Practice

Solve each equation. Check your solution.

- | | |
|---|--------------------------------|
| 1A. $3w + 2 = 7w$ | 1B. $5a + 2 = 6 - 7a$ |
| 1C. $\frac{x}{2} + 1 = \frac{1}{4}x - 6$ | 1D. $1.3c = 3.3c + 2.8$ |

2 Grouping Symbols

If equations contain grouping symbols such as parentheses or brackets, use the Distributive Property first to remove the grouping symbols.

TEKS A.5(A)

Example 2 Solve an Equation with Grouping Symbols

$$\begin{aligned} \text{Solve } 6(5m - 3) &= \frac{1}{3}(24m + 12). \\ 6(5m - 3) &= \frac{1}{3}(24m + 12) \\ 30m - 18 &= 8m + 4 \\ 30m - 18 - 8m &= 8m + 4 - 8m \\ 22m - 18 &= 4 \\ 22m - 18 + 18 &= 4 + 18 \\ 22m &= 22 \\ \frac{22m}{22} &= \frac{22}{22} \\ m &= 1 \end{aligned}$$

Original equation
Distributive Property
Subtract $8m$ from each side.
Simplify.
Add 18 to each side.
Simplify.
Divide each side by 22.
Simplify.

StudyTip

Solving an Equation
You may want to eliminate the terms with a variable from one side before eliminating a constant.

Guided Practice

Solve each equation. Check your solution.

2A. $8s - 10 = 3(6 - 2s)$

2B. $7(n - 1) = -2(3 + n)$

Some equations may have no solution. That is, there is no value of the variable that will result in a true equation. Some equations are true for all values of the variables. These are called **identities**.

TEKS A.5(A)

Example 3 Find Special Solutions

Solve each equation.

a. $5x + 5 = 3(5x - 4) - 10x$

$$\begin{aligned} 5x + 5 &= 3(5x - 4) - 10x && \text{Original equation} \\ 5x + 5 &= 15x - 12 - 10x && \text{Distributive Property} \\ 5x + 5 &= 5x - 12 && \text{Simplify.} \\ \underline{-5x} &= \underline{-5x} && \text{Subtract } 5x \text{ from each side.} \\ 5 &\neq -12 \end{aligned}$$

Since $5 \neq -12$, this equation has no solution.

b. $3(2b - 1) - 7 = 6b - 10$

$$\begin{aligned} 3(2b - 1) - 7 &= 6b - 10 && \text{Original equation} \\ 6b - 3 - 7 &= 6b - 10 && \text{Distributive Property} \\ 6b - 10 &= 6b - 10 && \text{Simplify.} \\ 0 &= 0 && \text{Subtract } 6b - 10 \text{ from each side.} \end{aligned}$$

Since the expressions on each side of the equation are the same, this equation is an identity. It is true for all values of b .

Guided Practice

3A. $7x + 5(x - 1) = -5 + 12x$

3B. $6(y - 5) = 2(10 + 3y)$

Go Online!



Use the algebra tiles **Virtual Manipulatives** in the eToolkit to visualize solving equations. If the mat has the same tiles on both sides at some point in the solution, the equation is an identity.



Concept Summary Steps for Solving Equations

Step 1 Simplify the expressions on each side. Use the Distributive Property as needed.

Step 2 Use the Addition and/or Subtraction Properties of Equality to get the variables on one side and the numbers without variables on the other side. Simplify.

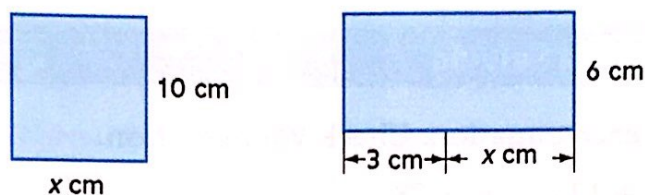
Step 3 Use the Multiplication or Division Property of Equality to solve.

There are many situations in which you must simplify expressions with grouping symbols in order to solve an equation.

TEKS A.5(A)

Example 4 Write an Equation

Find the value of x so that the figures have the same area.



A 3

C 6.5

B 4.5

D 7

Read the Item

The area of the first rectangle is $10x$, and the area of the second is $6(3 + x)$. The equation $10x = 6(3 + x)$ represents this situation.

Solve the Item

A $10x = 6(3 + x)$

B $10x = 6(3 + x)$

$10(3) \stackrel{?}{=} 6(3 + 3)$

$10(4.5) \stackrel{?}{=} 6(3 + 4.5)$

$30 \stackrel{?}{=} 6(6)$

$45 \stackrel{?}{=} 6(7.5)$

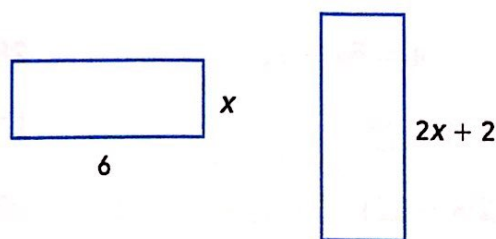
$30 \neq 36$ ✗

$45 = 45$ ✓

Since the value 4.5 results in a true statement, you do not need to check 6.5 and 7. The answer is B.

Guided Practice

4. Find the value of x so that the figures have the same perimeter.



F 1.5

G 2

H 3.2

J 4

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Check Your Understanding

Step-by-Step Solutions begin on page R13.

Go Online! Self-Check

Examples 1-3 Solve each equation. Check your solution.

TEKS A.5(A)

1. $13x + 2 = 4x + 38$

3. $6(n + 4) = -18$

5. $5 + 2(n + 1) = 2n$

7. $14v + 6 = 2(5 + 7v) - 4$

2. $\frac{2}{3} + \frac{1}{6}q = \frac{5}{6}q + \frac{1}{3}$

4. $7 = -11 + 3(b + 5)$

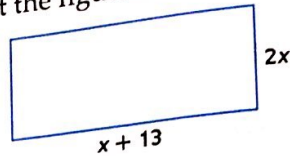
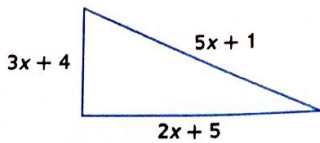
6. $7 - 3r = r - 4(2 + r)$

8. $5h - 7 = 5(h - 2) + 3$

Example 4

TEKS A.5(A)

9. MULTIPLE CHOICE Find the value of x so that the figures have the same perimeter.



A 4

B 5

C 6

D 7

Practice and Problem Solving

Extra Practice is on page R2.

Examples 1-3 Solve each equation. Check your solution.

TEKS A.5(A)

10. $7c + 12 = -4c + 78$

12. $9x - 4 = 2x + 3$

14. $\frac{b-4}{6} = \frac{b}{2}$

16. $8 = 4(r + 4)$

18. $5(g + 8) - 7 = 103$

20. $3(3m - 2) = 2(3m + 3)$

11. $2m - 13 = -8m + 27$

13. $6 + 3t = 8t - 14$

15. $\frac{5v-4}{10} = \frac{4}{5}$

17. $6(n + 5) = 66$

19. $12 - \frac{4}{5}(x + 15) = 4$

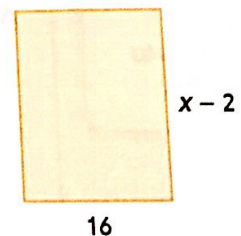
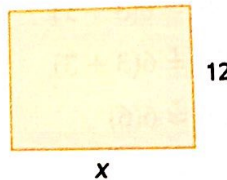
21. $6(3a + 1) - 30 = 3(2a - 4)$

Example 4

TEKS A.5(A)

22. GEOMETRY Find the value of x so the rectangles have the same area.

23. NUMBER THEORY Four times the lesser of two consecutive even integers is 12 less than twice the greater number. Find the integers.



24. MP ORGANIZE IDEAS Two times the least of three consecutive odd integers exceeds three times the greatest by 15. What are the integers?

Solve each equation. Check your solution.

25. $2x = 2(x - 3)$

27. $-5(3 - q) + 4 = 5q - 11$

29. $\frac{3}{5}f + 24 = 4 - \frac{1}{5}f$

31. $\frac{2m}{5} = \frac{1}{3}(2m - 12)$

33. $6.78j - 5.2 = 4.33j + 2.15$

35. $3.2k - 4.3 = 12.6k + 14.5$

26. $\frac{2}{5}h - 7 = \frac{12}{5}h - 2h + 3$

28. $2(4r + 6) = \frac{2}{3}(12r + 18)$

30. $\frac{1}{12} + \frac{3}{8}y = \frac{5}{12} + \frac{5}{8}y$

32. $\frac{1}{8}(3d - 2) = \frac{1}{4}(d + 5)$

34. $14.2t - 25.2 = 3.8t + 26.8$

36. $5[2p - 4(p + 5)] = 25$